LIST of ABSTRACTS OF THE MEETING:

PDE'S, SEMIGROUP THEORY AND INVERSE PROBLEMS

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The speakers' families are written in *italics*

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Wave front set of solutions to sums of squares of vector fields

Paolo Albano [1]

Abstract. We describe some results on analytic and Gevrey hypoellipticity for operators sums of squares of real analytic vector fields.

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Perturbation Method for Degenerate Inverse Problems in Banach Spaces

Mohammed AL Horani [1] and Angelo Favini [2]

Abstract. We are concerned with an inverse problem for a degenerate linear evolution equation. We begin with the first-order problem where both hyperbolic and parabolic cases will be considered. Also, a complete second-order differential inverse problem will be considered. We indicate sufficient conditions for existence and uniqueness of a solution. All the results apply well to inverse problems for equations from mathematical physics. As a possible application of the abstract theorems, some examples of partial differential equations are given.

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Modeling electrical conduction in biological tissues by equations with memory

Micol Amar [1]

Abstract. It is well known that electric potentials can be used in diagnostic devices to investigate the properties of biological tissues. Such techniques are essentially based on the possibility of determining the physiological properties of a living body by means of the knowledge of its electrical resistance. This leads to an inverse problem for the Laplace equation with real coefficients, which is the standard equation, when only a resistive behavior of the body is assumed. However, it has been observed that, applying high frequency alternating potentials to the body, a capacitive behavior takes place. This effect (known in physics as Maxwell-Wagner effect) is due to the electric polarization at the interface of the cell membranes, which act as capacitors, and it has been studied under the assumption that the biological tissue is modeled as a composite media with a periodic microscopic structure composed by two finely mixed phases (intra and extra cellular) separated by an imperfect interface (cellular membrane).

In this talk the homogenization theory is used in order to pass from the microscopic description to the macroscopic one, which is characterized by equations with memory. Moreover, the time exponential asymptotic stability of the solutions is studied, obtaining in the limit an elliptic equation with complex coefficients and providing in this way a theoretical justification to the phenomenological models currently used in electrical impedance tomography.

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On the Convergence Rate of the Glimm Scheme for General Hyperbolic Systems

Fabio Ancona [1]

Abstract. We consider the Cauchy problem for a general N-dimensional, strictly hyperbolic, quasilinear system $u_t + A(u)u_x = 0$, in one space variable, where no assumption is made on the eigenvalues of the matrix $A(u)$ besides requiring that they are real and distinct. We shall discuss how to construct a quadratic Glimm interaction functional which yields a sharp error estimate on the convergence rate of approximate solutions constructed by the Glimm scheme.

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Determination of source terms in degenerate parabolic equations

Piermarco Cannarsa [1]

Abstract. In their 1998 paper, Imanuvilov and Yamamoto proposed a new method to prove Lipschitz stability results for inverse source problems relative to uniformly parabolic equations. Such a method was based on global Carleman estimates, which are a priori estimates in weighted Sobolev norms for solutions of boundary value problems that are 'blind' to initial/terminal data. In this talk I will report on recent Lipschitz stability results for a class of degenerate parabolic equations, obtained in a joint work with Tort and Yamamoto for problems in one space dimensions and then extended to higher dimension in a joint paper with Martinez and Vancostenoble. We address both boundary and locally distributed observations using new degeneracy-adapted Carleman estimates and suitable Hardy-type inequalities.

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Identification problems for degenerate differential equations of hyperbolic type

Angelo Favini [1]

Abstract. Semigroups generated by multivalued linear operators and functional analytic techniques are used to solve some identification problems related to degenerate differential equations of the first order and of the second-order in Banach spaces.

The results are illustrated by some applications to inverse problems for degenerate partial differential equations.

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Recovering a scalar time dependent function in a multidimensional parabolic equation by a nonlocal boundary additional information.

U. Fedus [1] and A. Lorenzi [2]

Abstract. In this paper we solve (locally in time and under suitable assumptions on the data) an identification problem related to a linear parabolic equation when the additional information is *time-dependent and nonlocal in space*. More exactly, our problem consists in recovering a (positive) time-dependent coefficient β in front of the time derivative. We prove a local in time existence, uniqueness and stability result when the data belong to suitable function spaces.

Our basic tool is the Semigroup Theory of linear bounded operators.

We are concerned with recovering the unknown function $u : [0, T_0] \times \Omega \to \mathbb{R}$ and the coefficient $\beta \in C([0, T_0]; \mathbb{R}_+)$, for some $T_0 \in [0, T]$, in the following parabolic identification problem related to a bounded domain Ω in $\mathbb{R}^d$ of class C^2:

$$\beta(t)D_t u(t, x) = \mathcal{A}u(t, x) + A_1(t)u(t) + f_0(t, x), \quad (t, x) \in [0, T_0] \times \Omega, \tag{1}$$

$$u(0, x) = u_0(x), \quad x \in \Omega, \tag{2}$$

$$Bu(t, x) = Bu_0(x), \quad (t, x) \in [0, T_0] \times \partial\Omega, \tag{3}$$

$$\Phi[t, u(t, \cdot)] = h(t), \quad t \in [0, T_0]. \tag{4}$$

where $\mathcal{A}$ and $B(t)$, $t \in [0, T]$, denote, respectively, the following second-order and first-order linear differential operators in Ω:

$$\mathcal{A} = \sum_{j,k=1}^{d} a_{j,k}(x) D_{x_j} D_{x_k}, \quad A_1(t) = \sum_{j=1}^{d} a_j(t, x) D_{x_j} + a_0(t, x), \tag{5}$$

$$Bz(x) = b_0(x)z(x) + \sum_{j=1}^{d} b_j(x) D_{x_j} z(x), \quad x \in \Omega, \tag{6}$$

where $b_i \in C^1(\overline{\Omega})$ and $\left|\sum_{j=1}^{d} b_j(x) n_j(x)\right| \geq \mu_0 > 0, x \in \partial\Omega$. Functions $f_0 : [0, T] \times \Omega \to \mathbb{R}$, $u_0 : \Omega \to \mathbb{R}$ and $h : [0, T] \to \mathbb{R}$ are given and smooth, while $u \to \Phi[t, u]$, $t \in [0, T]$, is a linear functional, u being any (smooth) real-valued function defined on Ω.

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Partial reconstruction of a source term in a parabolic problem

Davide Guidetti [1]

Abstract. We consider a linear parabolic problem of the form

$$\begin{cases} D_t u(t, x, y) = A(t, x, D_x)u(t, x, y) + B(t, y, D_y) + g(t, x)f(t, x, y), \quad t \in [0, T], x \in U, y \in V, \\ u(0, x, y) = u_0(x, y), \quad x \in U, \ y \in V, \\ +(\textit{possibly}) \textit{ boundary condition in } \partial(U \times V), \end{cases} \tag{7}$$

in a cylindrical domain $U \times V$. $g(t, x)$ is unknown, together with u. We want to reconstruct both u and g, from a supplementary information of the form

$$\int_W u(t, x, y) d\mu(y),$$

with μ complex Borel measure in the closure $\overline{V}$ of V. We consider the case $U = \mathbb{R}^m$, $V = \mathbb{R}^n$, no boundary conditions, and A and B strongly elliptic of arbitrary (even) order, and the case of $m = 1$, $U = (0, 1)$, $A(t, x, D_x) = D_x^2$, $B(t, y, D_y) = B(y, D_y)$ of second order in V, with Dirichlet or first-order boundary conditions. We show results of existence and uniqueness of solutions (u, g).

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Strong solutions to certain fluid-solid interaction problems

Matthias Hieber [1]

Abstract. In this talk we consider the the movement of a rigid body in a fluid under the influence of gravitation. The fluid could be either of Newtonian or Non Newtonian or viscoelastic type. We consider the corresponding system of equations coupled with the balance laws for the momentum and the angular momentum and discuss various concepts of solutions. Note that the fluid-solid interface is a moving one and has to be found as part of the solution process.

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Inverse problems in semiconductors theory

Victor Isakov [1]

Abstract. We describe recent analytic and numerical results on identification of the conductivity coefficient of an elliptic equation which results from the system of partial differential equations modeling a semiconductor device. This equation corresponds to a physically important unipolar case. We derive dual problem, give a global uniqueness result for the so called doping profile (a curve dividing doped and undoped parts of the device) under some natural condition of the positivity condition of the flux across doping profile. In many practical situations, contrast between conductivity inside and outside doped region is very high. We derive asymptotics with respect to this contrast and state simplified asymptotic inverse problem. We give examples of efficient numerical reconstruction, based on single layer representation, and outline open problems and possible directions of future research.

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Study of Complete Abstract Elliptic Differential Equations In Non Commutative Cases

Maëlis Blot [1], Angelo Favini [2] and *Rabah Labbas*[3]

Abstract. In this work, we consider the following abstract second order complete elliptic differential equation

$$u''(x) + 2Bu'(x) + Au(x) - \omega u(x) = f(x), \quad x \in \mathbb{R},\ \omega > 0,$$

where f is an X-valued function on $\mathbb{R}$, $(X, \|.\|)$ being a complex Banach space and $(A, D(A))$, $(B, D(B))$ are two closed linear operators in X.

We give some new results and clarifications in the study of this equation on the whole line, extending those given by Favini, Labbas, Maingot, Tanabe and

Yagi in some recents papers. These authors have studied this equation on $[0, 1]$ with Dirichlet boundary conditions in the case of the space $C^\theta([0, 1]; X)$, $\theta \in]0, 1[$. They have proved existence, uniqueness and maximal regularity of the strict solution under some natural commutativity assumptions.

Here, we improve the study by developing a new non commutative approach in the space $BUC^\theta(\mathbb{R}; X)$, $\theta \in]0, 1[$. We will prove existence and uniqueness of a strict solution, that is, a function u such that

$$u \in BUC^2(\mathbb{R}; X) \cap BUC(\mathbb{R}; D(A)) \quad \text{and} \quad u' \in BUC(\mathbb{R}; D(B)),$$

and satisfying our equation. On the other hand we will establish the following maximal regularities property

$$u''(\cdot),\ Bu'(\cdot),\ Au(\cdot) \in BUC^{\theta}(\mathbb{R};X)\,.$$

The study is also considered in the case $L^p(\mathbb{R};X)$, $1 < p < \infty$, with an UMD Banach space X.

In the end, we will indicate the generalization of this study to the non commutative case of the same equation on $[0,1]$ with Dirichlet boundary conditions.

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Inverse problem for a structural acoustic interaction model

Irena Lasiecka [1]

Abstract Inverse problem of determining the potential caused by the source term entering an acoustic chamber is considered.

The PDE model consists of wave equation coupled to plate equation with the coupling occurring at the interface-manifold of lower dimension.

The source is being reconstructed from the measurements of acceleration of elastic wall, the latter being modeled by the plate equation.

The measurements are taken on the interface separating acoustic environment and the elastic wall.

Both uniqueness of reconstruction and stability estimates are established.

The proofs are based on recently developed Carleman's estimates applicable to Neumann unobserved boundaries along with sharp trace regularity results available for wave equations.

This is joint work with Shitao Liu.

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Approximate controllability and stabilization of parabolic integro-differential equations

Cătălin-George Lefter [1] and Alfredo Lorenzi [2]

Abstract. Let $\Omega \subset \mathbb{R}^N$ be an open set with a C^2-boundary. Let $\omega \subset\subset \Omega$ be an open subset. Consider an elliptic operator of the form

$$Ay = \sum_{i,j=1}^{n} \alpha_{ij} \frac{\partial^2 y}{\partial x_i \partial x_j} + \sum_{j=1}^{n} \beta_j \frac{\partial y}{\partial x_j} + \gamma y \tag{8}$$

where
i) $\alpha_{ij} \in C^1(\Omega)$, $\alpha_{ij} = \alpha_{ji}$, $i,j = 1,\dots,n$, and define a uniformly positive definite matrix:

$$\sum_{i,j=1}^{n} \alpha_{ij}(x)\xi^i \xi^j \geq C|\xi|^2,$$

for some positive constant C and all $x \in \Omega$, $\xi \in \mathbb{R}^N$.

ii) $\beta_i, \gamma \in L^\infty(\Omega)$.

Consider the controlled integro-differential equation

$$\begin{cases} y' = A(y(t) + (a * y)(t)) + Bu(t) \\ y(0) = y_0 \end{cases} \tag{9}$$

The memory kernel $a : [0, \infty) \to \mathbb{R}$ satisfies $a \in W^{1,p}_{loc}(0, T)$ for some $p \in (1, +\infty]$. The control operator $B : L^2(\omega) \to L^2(\Omega)$ is the extension by zero of functions defined on ω.

For a given $u \in \mathcal{U} := L^2(0, T, L^2(\omega))$ denote by $y^u(t)$ the solution to problem (9).

We study the following problems:

P1 Prove that under suitable conditions on the behaviour of a near 0, for given $T > 0$, the closure in $L^2(\Omega)$ of the set $\{y^u(T) : u \in \mathcal{U}\}$ is the whole space. This means that system (9) is *approximately controllable*.

P2 Find a feedback $u = Ky$ such that the feedback controlled system (9) is asymptotically stable, *i.e.* $y^{Ky(t)}(t) \to 0$ in $L^2(\Omega)$ for $t \to \infty$.

The main tool in approaching the above problems are the Carleman estimates for parabolic operators, with explicit constants, and a unique continuation result at initial time.

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Padé approximate impedance condition for thin shell.

Keddour Lemrabet [1]

Abstract. Many physical problems involve a body (Ω) coated by a thin shell $(\mathcal{U}_\delta)$ of small thickness δ (scatterer in wave propagation or stiffener in mechanics). One have to deal with a boundary and transmission problem (P_δ) set on a domain depending on the small parameter δ: (equation in Ω and equation in $\mathcal{U}_\delta$, transmission conditions at the interface Σ and boundary condition on $\partial\mathcal{U}_\delta \backslash \Sigma$.)

The effect of the thin shell $(\mathcal{U}_\delta)$ on the body (Ω) can be modelled by an impedance boundary condition (a linking between the Cauchy data) at the interface Σ. Problem (P_δ) is then reduced to a boundary value problem (Q_δ) set on the domain Ω (not depending on the small parameter δ): (equation in Ω and impedance condition on Σ).

The impedance condition on Σ depend on the parameter δ. To obtain the impedance operator modelling the effect of the thin shell, one have to solve a boundary value problem on $\mathcal{U}_\delta$ (and this is not an easy task). Since the parameter δ is small, one search approximate impedance condition of order n ($n = 0, 1, 2, 3$) and solve an approximate impedance boundary problem (Q^*_δ) : (equation in Ω,approximate impedance condition on Σ).

The most simple example in case $\mathcal{U}_\delta =]0, \delta[$ is when the equation in $]0, \delta[$ is $y''(t) + qy(t) = 0$ with $y'(\delta) = 0$.The impedance operator T_δ is given by $T_\delta(\varphi) = y'(0)$, where $y(t)$ is the solution satisfying $y(0) = \varphi$.

Approximations to the impedance T_δ are obtained through a Taylor expansion of $y'(\delta)$. Writing $y'(\delta) = y'(0) + \delta y''(0) + \left(\delta^2/2\right) y^{(3)}(0) + ...$ and then using the equation to calculate the derivatives of order $2, 3, ...$(in terms of $y'(0)$ and $y(0) = \varphi$), one gets the approximate impedance operator (of order 3) $T^*_\delta = -\delta \left[1 + \left(\delta^2/2\right) q\right]^{-1} q \left[1 + \left(\delta^2/6\right) q\right]$.

In the general case of a thin shell, one uses the local coordinate system (tangent plane to Σ and the normal to Σ, $(m, t) \in \Sigma \times]0, \delta[$) to write the quation on the shell $\mathcal{U}_\delta$ as an abstract

differential system on $]0, \delta[$: $\mathcal{Y}'(t) = \mathcal{M}(t)\mathcal{Y}(t)$. For fixed t, $\mathcal{Y}(t)$ is a function of m and $\mathcal{M}(t)$ is a differential operator in the tangential variable m.

For example for the Laplace equation $\Delta u = 0$ on $\mathcal{U}_\delta$ with the boundary condition $\frac{\partial u}{\partial n} = 0$ on $\mathcal{U}_\delta \backslash \Sigma$, one set $\mathcal{Y}(t)(m) = \left(u(t,m), \frac{\partial}{\partial t}u(t,m)\right)$ and write the the operator Δ in terms of $\frac{\partial}{\partial t}$ and the tangential gradient ∇_Γ. A Taylor expansion (at order 2) of $\mathcal{Y}(\delta)$ at 0 and the boundary condition $\frac{\partial}{\partial t}u(\delta,m) = 0$ lead to the Padé approximate condition $\frac{\partial}{\partial t}u(0,m) = T^*_\delta u(0,m)$ where T^*_δ is given by ($\mathcal{C}$ and $\mathcal{H}$ being the curvature operator and the mean curvature of Σ) $T^*_\delta = \delta\left[I - \left(\delta^2/2\right)\nabla_\Gamma.\nabla_\Gamma\right]^{-1}\nabla_\Gamma.\left[I - (\delta/2)\left(\mathcal{C} - 2\mathcal{H}\right)\right]\nabla_\Gamma$.

The same techniques apply to give approximate impedance of Pade-type up to order 3 for thin scatterer for Hemholtz equation and harmonic Maxwell system and thin stiffener for the Lamé elasticity system.

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Severely ill-posed linear integrodifferential problems

Alfredo Lorenzi [1]

Abstract. Uniqueness and continuous dependence results for some severely ill-posed linear integrodifferential problems are deduced via the same Carleman estimates used to deal with Differential Control problems.

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Compactness and invariance properties of evolution operators associated with Kolmogorov operators with unbounded coefficients

Luca Lorenzi [1]

Abstract. In this talk we consider nonautonomous elliptic operators $\mathcal{A}$ with nontrivial potential term defined in $I \times \mathbb{R}^d$, where I is a right-halfline (possibily $I = \mathbb{R}$). We prove that we can associate an evolution operator $(G(t,s))$ with $\mathcal{A}$ in the space of all bounded and continuous functions on $\mathbb{R}^d$. We also study the compactness of the operator $G(t,s)$. Finally, we provide sufficient conditions guaranteeing that each operator $G(t,s)$ preserves the usual L^p-spaces and $C_0(\mathbb{R}^d)$.
This is a joint paper with Luciana Angiuli, department of Mathematics of the Università del Salento.

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Abstract Differential Equations of Elliptic Type with Robin Boundary Conditions, in Hölder Spaces

Mustapha Cheggag [1], Angelo Favini [2], Rabah Labbas [3], *Stéphane Maingot* [4] (stephane.maingot@univ-lehavre.fr) and Ahmed Medeghri [5]

Abstract. We consider the second order abstract differential equation in a complex Banach space X

$$u''(x) + 2Bu'(x) + Au(x) = f(x), \quad x \in (0,1), \tag{10}$$

together with the abstract Robin boundary conditions

$$hu'(0) - Hu(0) = d_0, ku'(1) + Ku(1) = d_1, \tag{11}$$

where d_0, d_1 are given elements of X and A, B, h, H, k, K are closed linear operators in X.

A. Favini, R. Labbas, S. Maingot, H. Tanabe and A.Yagi studied this Problem when (11) reduces to the Dirichlet boundary conditions $u(0) = d_0, u(1) = d_1$ and when

$$f \in L^p(0,1;X), 1 < p < +\infty \text{ or } f \in C^\theta([0,1];X), 0 < \theta < 1.$$

M. Cheggag, A. Favini, R. Labbas, S. Maingot et A. Medeghri, gave, in the L^p case, positive results for Problem (10)-(11) with $h = K = I$ and $B = k = 0$, that is Problem

$$\left\{ \begin{array}{l} u''(x) + Au(x) = f(x), \quad x \in (0,1), \\ u'(0) - Hu(0) = d_0, \;\; u(1) = d_1. \end{array} \right. \tag{12}$$

In the following we consider the case $f \in C^\theta([0,1];X)$, $0 < \theta < 1$ and study Problem (12) under some assumptions on operators A and H.

Our results are the following:

- We give necessary and sufficient conditions on the data d_0, d_1 to obtain a unique strict solution u to Problem (12). We study also, for the strict solution u, the maximal regularity property u'', $Au \in C^\theta([0,1];X)$.
- We study various situations in which our hypotheses on A and H are satisfied.
- We apply our abstract results to partial differential equations.

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Identification problems for semilinear integrodifferential hyperbolic equations with transformed arguments

Alfredo Lorenzi [1] and *Francesca Messina* [2]

Abstract. This work was inspired by the article by A. M. Denisov (2008), where the author studied a one-dimensional semilinear hyperbolic problem related to the wave equation with a nonlinear term with a transformed argument, a function α depending on time. The author's aim was to determine the unknown function $\alpha(t)$ by using an additional information depending on time, at a particular point x_0, $u(t, x_0) = b(t)$.

The aim of the present paper is to generalize such a result under several aspects: (i) the equation is a general semilinear integrodifferential hyperbolic d–dimensional equation in divergence form ($d = 1, 2, 3$); (ii) the space–time set considered here is a smooth cylinder where surface boundary conditions are prescribed; (iii) the term with transformed arguments is allowed to contain integral operators; (iv) the additional information is of integral type.

In particular we consider the following semilinear integro-differential Cauchy problem

$$D_t^2 u(x,t) - A(x,D_x)u(x,t) = f_1(t,x,u(x,t),u(x,\alpha(t)))$$

$$+\int_\Omega f_2(t,x,y,u(y,t),u(y,\alpha(t)))\,dy, \quad (x,t)\in\Omega\times[0,T], \tag{13}$$

$$u(x,0)=v_0(x), \quad D_t u(x,0)=v_1(x), \quad x\in\Omega, \tag{14}$$

$$Bu(x,t)=0, \quad (x,t)\in\partial\Omega\times[0,T]. \tag{15}$$

Here Ω is a bounded connected open set in $\mathbb{R}^d$, $d=1,2,3$, with a C^2-boundary, $\mathbf{n}(x)=(n_1(x),\ldots,n_d(x))$, while the operators A, B are defined by

$$A(x,D_x)=\sum_{j=1}^{d} D_{x_j}[a_{j,k}(x)D_{x_k}]+a_{0,0}(x), \tag{16}$$

$$B(u)(x):=B_0(u)(x)=u(x), \quad \text{or} \quad B(u)(x):=B_1(u)(x)=\sum_{j=1}^{d}\nu_k(x)D_{x_k}u(x), \quad x\in\partial\Omega, \tag{17}$$

where $\nu(x)=(\nu_1(x),\ldots,\nu_d(x))$ denotes the conormal vector related to $A(x,D_x)$, i.e.

$$\nu_k(x)=\sum_{j=1}^{d} n_j(x)a_{j,k}, \quad x\in\partial\Omega,\ k=1,\ldots,d. \tag{18}$$

Moreover α is a function in $C^1([0,T])$ such that $\alpha(0)=0$ and $0<\alpha'(t)\le 1$, for all $t\in[0,T]$. To recover the unknown function α, we introduce the additional information

$$\Phi[u(x,t)] =: \int_\Omega \varphi(x)u(x,t)\,dx = h(t), \quad t\in[0,T], \tag{19}$$

where $\varphi:\Omega\to\mathbb{R}$ and $h:[0,T]\to\mathbb{R}$ are given functions. Along with the specific integro–differential equation (13) we have considered a more general second–order in time operator equation in a Hilbert space, with a term contained a transformed argument.

Under a suitable general additional information we will determine (*locally in time*) the unknown function α in the considered general case. This result will be applied to our specific hyperbolic integro-differential problem (13)–(17), (19).

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Elliptic operators with unbounded diffusion coefficients in L^p

Giorgio Metafune [1] and C. Spina [2]

Abstract. In this talk we focus our attention on elliptic operators with unbounded diffusion of the form

$$Lu=(1+|x|^\alpha)\Delta u, \tag{20}$$

for positive values of α, on $L^p=L^p(\mathcal{R}^N,dx)$ with respect to the Lebesgue measure. The case $\alpha\le 2$ has been already investigated by S. Fornaro and L. Lorenzi who proved that the operator

above generates a strongly continuous and analytic semigroup in L^p and in spaces of continuous functions. For $1 < p < \infty$ an explicit description follows from the a-priori estimates

$$\|(1+|x|^\alpha)D^2u\|_p \leq C(\|u\|_p + \|(1+|x|^\alpha)\Delta u\|_p.$$

Similar estimates hold for a mor general class of operators. They can be deduced by some weigthed norm inequalities for Caldéron-Zygmund singular integrals. Muckenhoupt and Wheeden for example proved that estimates of the form

$$\|aD^2u\|_p \leq C\|a\Delta u\|_p$$

are true for weights a in suitable Muckenhoupt classes. In particular the estimates above imply that

$$\||x|^\alpha D^2u\|_p \leq C\||x|^\alpha \Delta u\|_p \tag{21}$$

and

$$\|(1+|x|^\alpha)D^2u\|_p \leq C(\|u\|_p + \|(1+|x|^\alpha)\Delta u\|_p$$

for $0 < \alpha < N/p'$ where p' is the conjugate exponent of p.
Similar estimates follow also by the work of Kree who studied singular integrals in L^p spaces with respect to the weight $1+|x|^\alpha$, $-N/p' < \alpha < N/p'$.
We will prove that for $2 < \alpha \leq (N-2)(p-1)$ and $N \geq 3$ the operator above generates a semigroups in L^p which is analytic when for $\alpha < (N-2)(p-1)$. Moreover for $2 < \alpha < N/p'$ an explicit description of the domain follows from an improved version of the a-priori estimates (21).

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The Cahn-Hilliard equation with dynamic boundary conditions

Alain Miranville [1]

Abstract. Our aim in this talk is to discuss the Cahn-Hilliard equation in phase separation associated with dynamic boundary conditions. Such boundary conditions have been introduced by physicists in order to account for the interactions with the walls in confined systems.

We will focus on the case of singular potentials (having in mind the thermodynamically relevant logarithmic potentials) and will in particular discuss issues related with the existence of (classical) solutions.

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Recovering a positive constant in linear evolution equations in Hilbert spaces

Gianluca Mola [1]

Abstract. Let H be a real Hilbert space and $A : \mathcal{D}(A) \to H$ be an unbounded operator which admits power A^σ of exponent $\sigma \in [0,1)$. We consider the identification problem consisting in searching for a function $u : [0,T] \to H$ and a constant $\mu > 0$ solving the initial-value problem

$$\partial_t u + Au = \mu\, A^\sigma u, \quad t \in [0,T], \quad u(0) = u_0,$$

and the additional condition

$$\alpha\|u(T)\|^2 + \beta\int_0^T \|A^{1/2}u(\tau)\|^2 d\tau = \rho,$$

where $u_0 \in H$ and $\alpha, \beta, \rho > 0$ are given. By means of a finite-approximation scheme, we prove the existence of a unique solution (u, μ) of suitable regularity on the whole interval $[0, T]$, and exhibit an explicit continuous dependence estimates of Lipschitz-type on the data $u_0 \in H$ and $\alpha, \beta, \rho > 0$, provided that the latter fulfill an priori bound. Also, we provide specific applications on second and fourth-order parabolic initial-boundary value problems.

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Remarks on linear Schrödinger evolution equations with Coulomb potential with moving center

Noboru Okazawa (okazawa@ma.kagu.tus.ac.jp), Tomomi Yokota and Kentarou Yoshii

Abstract. This paper is concerned with Cauchy problems for the linear Schrödinger evolution equation:

$$i\frac{\partial u}{\partial t}(x,t) + \Delta u(x,t) + \frac{1}{|x-a(t)|}u(x,t) + V_1(x,t)u(x,t) = f(x,t)$$

in $\mathbb{R}^N \times [0,T]$, subject to initial condition: $u(x,0) = u_0(x) \in H^2(\mathbb{R}^N) \cap H_2(\mathbb{R}^N)$, where $i := \sqrt{-1}$, $a : [0,T] \to \mathbb{R}^N$ expresses the center of the Coulomb potential, V_1 and $f : [0,T] \times \mathbb{R}^N \to \mathbb{R}$ are another potential and an inhomogeneous term while $H_2(\mathbb{R}^N) := \{v \in L^2(\mathbb{R}^N); |x|^2 v \in L^2(\mathbb{R}^N)\}$. The strong formulation of this problem (with $f \equiv 0$) has been solved by Baudouin-Kavian-Puel (2005) partly with formal computation. In this paper we reconstruct their argument with rigorous proofs. Moreover, a solution u satisfies the energy estimate

$$\left\|\frac{\partial u}{\partial t}(t)\right\| + \|u(t)\|_{H^2\cap H_2} \le C_0(\|u_0\|_{H^2\cap H_2} + \|f\|_F),$$

where $C_0 > 0$ is a constant depending on a, V_1 and T, while $\|f\|_F$ is the sum of some norms of f.

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Temperature and flux controllability for heat equations with memory

Luciano Pandolfi [1]

Abstract. In the case of the standard heat equation, the flux at a certain time is a multiple of the gradient of the temperature and so it is known once the temperature is known at the same time. So, it is not possible to control independently the temperature and the flux.

Things are different in the case of the heat equation with memory,

$$\theta_t(t,x) = \int_0^t N(t-s)\Delta\theta(s,x)\,\mathrm{d}s\,.$$

In this case the flux is a multiple of

$$\int_0^t N(t-s)\nabla\theta(s,x)\,\mathrm{d}s$$

and temperature and flux are only mildly related. Hence, it makes sense to study at what extent they can be independently controlled.

In this talk we consider heat equations with memory in one space dimension. We use a moment type approach with respect to a sequence of functions especially related to the heat equation with memory under study and we prove that heat and flux can be independently controlled, both in the space of square integrable functions.

The result has been obtained together with S. Avdonin.

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Exponential stability of the strongly damped wave equation with boundary feedback laws with delay

Cristina Pignotti [1]

Abstract. We consider a stabilization problem for the wave equation with structural damping and boundary feedback laws with a time delay. We will give an exponential stability estimate under a suitable relation between the internal damping and the boundary laws. This result is based on an identity with multipliers that allows to obtain a uniform decay estimate for a suitable energy functional.

Moreover, we will present some instability examples when this condition is not satisfied.

Joint work with Serge Nicaise (Université de Valenciennes).

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Degenerating PDE's for phase transitions in thermoviscoelstic materials

Elisabetta Rocca [1]

Abstract. This is a joint work with Riccarda Rossi (University of Brescia, Italy).

We address the analysis of a nonlinear and degenerating PDE system, proposed by M. Frémond for modelling phase transitions in viscoelastic materials subject to thermal effects. The system features an internal energy balance equation, governing the evolution of the absolute temperature θ, an evolution equation for the phase change parameter χ, and a stress-strain relation for the displacement variable $\mathbf{u}$. The main novelty of the model is that the equations for χ and $\mathbf{u}$ are coupled in such a way as to take into account the fact that the properties of the viscous and of the elastic parts influence the phase transition phenomenon in different ways. However, this brings about an elliptic degeneracy in the equation for $\mathbf{u}$ which needs to be carefully handled.

In [E. R., R. Rossi: *Analysis of a nonlinear degenerating PDE system for phase transitions in thermoviscoelastic materials*, J. Differential Equations, **245** (2008), 3327–3375] we first prove a (*local in time*) well-posedness result for (a suitable initial-boundary value problem for) the above mentioned PDE system, in the (spatially) three-dimensional setting. Secondly, in [E. R., R. Rossi: *Global existence of strong solutions to the one-dimensional full model for phase transitions in thermoviscoelastic materials*, Appl. Math., **53** No. 5 (2008), 485–520] we restrict to the one-dimensional case, in which, for the same initial-boundary value problem, we indeed obtain a *global* well-posedness theorem and we perform a long-term dynamics analysis on the system.

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Continuous dependence on dynamical boundary conditions of Wentzell type

Silvia Romanelli [1]

Abstract. Let Ω be a bounded domain in $\mathcal{R}^N$ with a C^2 boundary. Let $\mathcal{A}(x) = (a_{ij}(x))$ be a real hermitian $N \times N$ matrix with coefficients in $C^1(\overline{\Omega})$ which is uniformly positive definite, that is,

$$\mathcal{A}(x)\xi \cdot \xi = \sum_{i,j=1}^{N} a_{ij}(x)\xi_i\xi_j \geq \alpha_0|\xi|^2$$

for some $\alpha_0 > 0$ and all $\xi \in \mathcal{R}^N$, $x \in \overline{\Omega}$. For $u = u(x,t)$ and $q = 0$, the problem

$$\frac{\partial u}{\partial t} = \nabla \cdot \mathcal{A}(x)\nabla u \quad (= \sum_{i,j=1}^{N} \partial_i(a_{ij}(x)\partial_j u)), \tag{22}$$

$$u(x,0) = f(x), \tag{23}$$

$$\nabla \cdot \mathcal{A}(x)\nabla u + \beta(x)\partial_n^{\mathcal{A}} u + \gamma(x)u - q\beta(x)\Delta_{LB}u = 0 \tag{24}$$

is governed by a quasicontractive (C_0) semigroup on X_p, $1 \leq p \leq \infty$ (analytic for $1 < p < \infty$), where $X_\infty = C(\overline{\Omega})$ and $X_p = L^p(\Omega, dx) \oplus L^p(\partial\Omega, \frac{dS}{\beta})$ for $1 \leq p < \infty$; here $(x,t) \in \Omega \times [0,\infty)$ in (1), $x \in \Omega$ in (2), $(x,t) \in \partial\Omega \times [0,\infty)$ in (3), $\beta, \gamma \in C^1(\partial\Omega)$ are real, $\beta > 0$ on $\partial\Omega$, $q \in [0,\infty)$, Δ_{LB} is the Laplace-Beltrami operator on $\partial\Omega$, and

$$\partial_n^{\mathcal{A}} u = \mathcal{A}\nabla u \cdot n = \sum_{i,j=1}^{N} a_{ij}(x)n_i(\partial_j u) \tag{25}$$

is the conormal derivative of u with respect to $\mathcal{A}$, n being the unit outer normal at $x \in \partial\Omega$. According to [1], the solution of (1)-(3) depends continuously on (β, γ, q). Analogous problems will be presented in the context of the wave equation with Wentzell boundary condition (3) in X_2, as in the joint paper [2] with T. Clarke, G.R. Goldstein and J.A. Goldstein. Note that condition (3) can be interpreted as a dynamic boundary condition.

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The Cahn-Hilliard Equation with Nonpermeable Walls

Gisèle Ruiz Goldstein [1]

Abstract. We discuss the problem of phase separation for a binary mixture in a domain with nonpermeable walls in the context of the Cahn-Hilliard model. The nonpermeability of the boundary leads to consideration of a new boundary condition for the chemical potential. Dynamic boundary conditions for the relative concentration are considered. The derivation of the model as well as a discussion of the solution of this problem will be presented.

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Uniqueness and stability for inverse problems for second-order hyperbolic equations

Roberto Triggiani [1]

Abstract. We consider some inverse problems such as recovering the damping coefficient of a wave equation with Neumann Boundary conditions from one measurement consisting of the Dirichlet trace on a suitable part of the boundary.

These two issues are actually related to 'observability' and 'continuous observability' problems in control theory. The solution given here - initially on an euclidean domain - is based on (i) sharp Carleman estimates and (ii) continuous observability estimates obtained for very general second order hyperbolic equations in Lasiecka-Triggiani-Zhang (AMS Contemporary Mathematics, 2000) plus a final step proposed in V. Isakov's book on Inverse problems (2nd edition).

The procedure can be extended to second order hyperbolic equations defined on a finite dimensional Riemannian manifold with boundary; in particular, second order hyperbolic equations on an euclidean domain with variable coefficient principal part. Here the needed (i) sharp Carleman estimates and (ii) continuous observability estimates are invoked from Triggiani-Yao, AMO, 2002.

Time permitting, we shall also consider inverse problems for coupled hyperbolic equations; as well as the counterpart of all these inverse problems for Schrodinger equations.

This is joint work by the speaker with his PhD student Shitao Liu.

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On the heat and the Laplace equations with dynamical boundary conditions of reactive–diffusive type

Enzo Vitillaro [1]

Abstract. The aim of this talk is to present some recent results obtained in collaboration with Juan Luis Vazquez of the Universidad Autonoma de Madrid concerning the well–posedness of the Laplace equation and the heat equation with reactive–diffusive dynamical boundary conditions. More precisely we shall consider the two problems

$$\begin{cases} \Delta u = 0 & \text{in } (0,\infty)\times\Omega, \\ u_t = k u_\nu + l\Delta_\Gamma u & \text{on } (0,\infty)\times\Gamma, \\ u(0,x) = u_0(x) & \text{on } \Gamma, \end{cases} \tag{26}$$

and

$$\begin{cases} u_t - \Delta u = 0 & \text{in } (0,\infty)\times\Omega, \\ u_t = k u_\nu + l\Delta_\Gamma u & \text{on } (0,\infty)\times\Gamma, \\ u(0,x) = u_0(x) & \text{on } \Omega, \end{cases} \tag{27}$$

where $u = u(t,x)$, $t \geq 0$, $x \in \Omega$, $\Delta = \Delta_x$ denotes the Laplacian operator with respect to the space variable, Ω is a C^∞ regular domain, Δ_Γ denotes the Laplace–Beltrami operator on Γ, ν

the outward normal to Ω, and k and l are given positive constants. The analysis of Problem (1) is used in the study of Problem (2).

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An Identification Problem for a Semilinear Evolution Equation in a Banach Space

Alfredo Lorenzi [1], *Ioan I. Vrabie* [2]

Abstract. Theorem. *Let X be a real Banach space, let $A : D(A) \subseteq X \to X$ be the infinitesimal generator of a compact C_0-semigroup of contractions, $\{S(t);\ t \geq 0\}$, let $\xi_0, \xi_1 \in D(A)$, let $f : [0,1] \to X$ be a C^1-function and let $\varphi : X \to \mathbb{R}_+$ be a C^1-functional for which there exists $L > 0$ such that its Fréchet derivative calculated at $u \in X$, i.e. $\varphi'(u)$, satisfies $\|\varphi'(u)\|_{X^*} \leq L$, uniformly for $u \in X$. Let $C^0(\xi_0, \xi_1) = \Big\{C([0,1];X);\ u(0) = \xi_0,\ \int_0^1 u(t)\,dt = \xi_1\Big\}$, and let $\xi_0, \xi_1 \in D(A)$ be such that the family of operators $\big\{T_u;\ u \in C^0(\xi_0, \xi_1)\big\}$, defined by*

$$T_u z = \left\{ \int_0^1 \varphi(u(s))[I - S(1-s)]\,ds \right\} z \tag{28}$$

for $z \in X$, is uniformly invertible, i.e., for each $u \in C^0(\xi_0, \xi_1)$, we have $T_u(X) = X$, and there exists $\gamma > 0$ such that

$$\gamma \left\| \left\{ \int_0^1 \varphi(u(s))[I - S(1-s)]\,ds \right\} z \right\| \geq \|z\| \tag{29}$$

for each $u \in C^0(\xi_0, \xi_1)$ and each $z \in X$. Finally, let us assume that

$$\gamma L \left\| S(1)\xi_0 - \xi_0 - A\xi_1 - \int_0^1 S(1-s)f(s)\,ds \right\| < 1. \tag{30}$$

Then, there exists a strict solution (u, z) to the identification problem

$$\begin{cases} u'(t) = Au(t) + f(t) + \varphi(u(t))z, \quad t \in (0,1), \\ u(0) = \xi_0 \\ \int_0^1 u(t)\,dt = \xi_1. \end{cases} \tag{31}$$

admitting the implicit representation

$$u(t) = S(t)\xi_0 + \int_0^t S(t-s)f(s)\,ds + \int_0^t S(t-s)\varphi(u(s))z\,ds \tag{32}$$

$$z = T_u^{-1}\Big[S(1)\xi_0 - \xi_0 - A\xi_1 - \int_0^1 S(1-s)f(s)\,ds\Big]. \tag{33}$$

Under additional conditions, the solution given by the previous Theorem is unique and continuously depends on the data. An illustrative example is also included.

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Elliptic Differential-Operator Problems with Unbounded Operator Boundary Conditions

Yakov Yakubov [1]

Abstract. In a UMD Banach space E, we consider a boundary value problem for a second order elliptic differential-operator equation with a spectral parameter when one of the boundary conditions, in the principal part, contains a linear unbounded operator in E. A theorem on an isomorphism is proved and an appropriate estimate of the solution with respect to the space variable and the spectral parameter is obtained. Moreover, discreteness of the spectrum and completeness of a system of root functions corresponding to the homogeneous problem are established. Finally, applications of obtained abstract results to nonlocal boundary value problems for elliptic differential equations with a parameter in non-smooth domains are given.

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